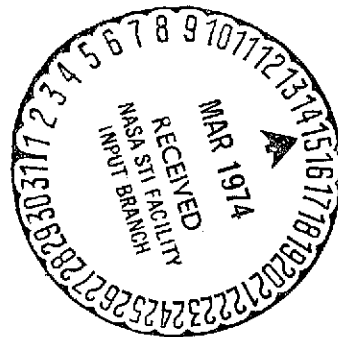


SOLAR MAGNETIC FIELD MEASUREMENTS

E. H. Schröter

Translation of "Solare Magnetfeldmessungen,"
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16. Abstract The article discusses problems in deriving the magnetic field in a region of solar activity, rather than the results of solar magnetic field measurements. The Stokes vector and its relationship to the magnetic field vector are described. Sources of error apart from those already known for the still atmosphere are outlined: nonvalidity of hydrostatic equilibrium, scattering, magneto-optical effects, and errors in assumptions about magnetic fields as functions of depth. Existing polarimeters are described, as well as possible vector polarimeters. Estimates are made on detection limits for solar magnetic fields.			
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SOLAR MAGNETIC FIELD MEASUREMENTS

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By now it is clear that all manifestations of solar activity/55* such as spots, flares, protuberances, eruptions, etc., are dictated by the spatial structure of the local solar magnetic fields and the changes in them. It is thus understandable that the measurements of magnetic fields in the regions of solar activity has in the past 60 years¹ become one of the focal points of solar physics. In the last two decades, a much greater effort can be observed (see e.g. the IAU Symposium No. 43 in Paris, 1971) on both the experimental apparatus and on the theoretical interpretation of the measured data. However, the increased precision requirements on the results -- a precision necessary for the understanding of manifestations of solar activity -- can by only partially satisfied. It thus seems appropriate not to report on the results of solar magnetic field measurements at this point, but instead to outline the problems and difficulties which hamper the derivation of the spatial structure of the magnetic field vector in a region of solar activity.

Stokes Vector and Magnetic Vector (Theory of Line Formation in Magnetic Field)

The measurements are based on the Zeeman effect in absorption. Fig. 1 depicts its most important properties: the splitting of the Fraunhofer lines into several components (for a Zeeman triplet, into two σ -components and the unshifted /56

¹ On June 25, 1908, G. E. Hale at the Mt. Wilson Observatory succeeded in detecting the magnetic field in sunspots for the first time.

* Numbers in the margin indicate pagination in the foreign text.

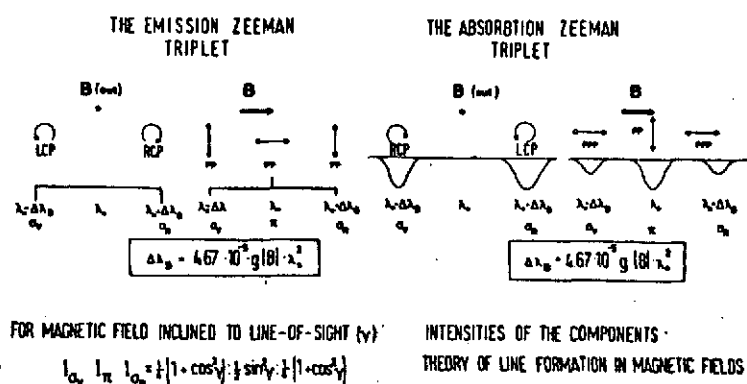


Fig. 1.

field vector and line-of-sight), and ϕ (azimuth of the field with the zero direction yet to be chosen), then with an arbitrary angle γ , the two σ -components are partially elliptically polarized with opposite senses of rotation, and the π -component is linearly polarized. The ratio of the axes of the polarization ellipse is determined by the angle γ , and the azimuth of the major axis of the ellipse by the angle ϕ . The splitting

$$\Delta\lambda_B = |\lambda_\pi - \lambda_\sigma|$$

yields $|B|$.

The finite width of the solar lines means that the splitting $\Delta\lambda_B$ can be directly measured only for fields $|B| \geq 1500$ Gauss. To determine smaller fields and to measure the field vector, the polarization state within a magnetically splitting Fraunhofer line must be analyzed.

As an aid in the representation (and mathematical treatment) of the polarization state of light, the so-called Stokes vector suggests itself (Shurcliff, 1962). If I = the total intensity of the partially polarized light, V = the intensity of the

π -component) and modification of the polarization state within the line.

If the magnetic field vector \underline{B} is characterized by the three field components $|B|$ (absolute magnitude of the field in Gauss), γ (angle between the

circularly polarized component (with the appropriate signs for left and right circular polarization), and Q and U two further parameters with the dimensions of intensity which describe the linear-polarization component $\sqrt{Q^2 + U^2} = P_{\text{lin}}$, then the polarization state of the light is completely described by the vector (I, Q, U, V). Illustrations for complete polarization:

$\rightarrow: (1, 1, 0, 0)$, $\uparrow: (1, -1, 0, 0)$, $\nearrow: (1, 0, 1, 0)$, $\searrow: (1, 0, -1, 0)$, $\nearrow: (1, 0, 0, 1)$, $\searrow: (1, 0, 0, -1)$.

For complete polarization, $I^2 = Q^2 + U^2 + V^2$, while for partial polarization $I^2 > Q^2 + U^2 + V^2$, and the degree of polarization P is defined by $P = ((Q^2 + U^2 + V^2)/I^2)^{1/2}$.

By analogy to the definition of the relative line absorption coefficient $\eta_\lambda = \kappa_L/\kappa_K$ for unpolarized radiation, the corresponding absorption coefficients η_I , η_Q , η_V , η_U can be defined by the Stokes parameters. The Sears formulas then state:

$$\begin{aligned}\eta_I &= \frac{1}{2} \eta_P \sin^2 \gamma + \frac{1}{4} (1 + \cos^2 \gamma) (\eta_R + \eta_B) \\ \eta_Q &= [\frac{1}{2} \eta_P - \frac{1}{4} (\eta_R + \eta_B)] \sin^2 \gamma \cos 2\varphi \\ \eta_U &= [\frac{1}{2} \eta_P - \frac{1}{4} (\eta_R + \eta_B)] \sin^2 \gamma \sin 2\varphi \\ \eta_V &= \frac{1}{2} (\eta_R - \eta_B) \cos \gamma\end{aligned}\tag{1)}$$

where

$$\eta_P = \eta_\lambda(\lambda) \quad \eta_R = \eta_\lambda(\lambda + \Delta\lambda_R) \quad \eta_B = \eta_\lambda(\lambda - \Delta\lambda_B).$$

Assuming that all three Zeeman components are optically thin (i.e. $I_P \sim \eta_P$, $I_{GR} \sim \eta_R$, etc.), the relationships in (1) provide the first rough relationship between the Stokes vector (in this case, $\eta_I \rightarrow I$, $\eta_Q \rightarrow Q$, $\eta_U \rightarrow U$, and $\eta_V \rightarrow V$) and the magnetic field vector. It is convenient to choose the coordinate system for describing the Stokes vector so that $U = 0$, i.e. $\sin 2\phi = 0$ and $\cos 2\phi = 1$ (Fig. 2). Fig. 3 summarizes in simplified form the principle of such calculations, compares it with the corresponding calculations for unpolarized radiation, and collects the figures which are involved in the calculations. For details of this

STOKES - PARAMETERS WITHIN A FRAUNHOFER LINE WITH NORMAL PATTERN

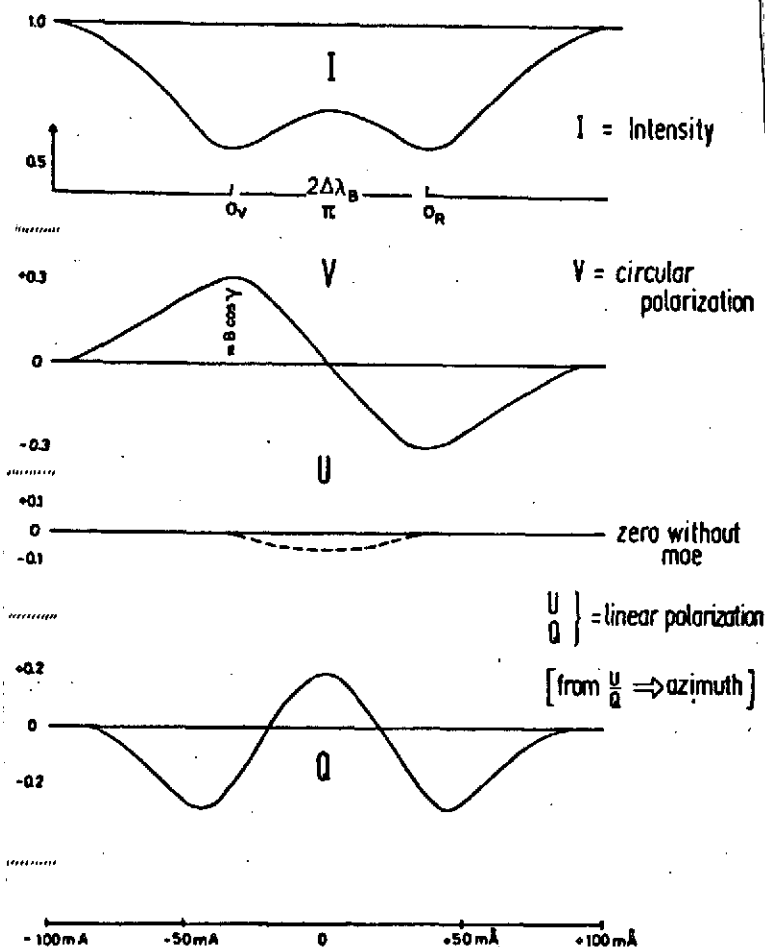


Fig. 2.

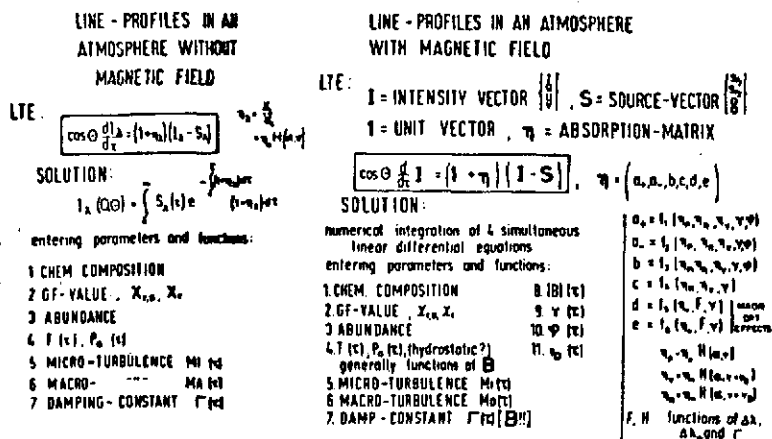


Fig. 3.

theory, we refer the reader to the survey article of Stenflo (1971). Our concern is the group of additional error sources beyond the known uncertainty factors in a still atmosphere:

a) For every subdivision of the region of activity (umbrae, penumbrae, flares, etc.), the precise stratification of temperature and pressure must be known; as is known, the question of whether the hydrostatic equilibrium holds in these solar regions had not been cleared up. If hydrostatic equilibrium is not guaranteed (which, strictly speaking, is always the case), there is a direct coupling between $P_G(\tau)$ and $B(\tau)$, and $P_G(\tau)$ would now have to be calculated for an assumed $B(\tau)$ via the fundamental magneto-hydrodynamic equations.

b) If coherent and incoherent scattering are to play a role /58 in line formation, the calculations become still more complicated, and more free parameters enter. Attempts at allowing for scattering processes have already been made (with regard to the literature, see Stenflo (1971)). Hyder (1968) further indicated that ignoring the "Hanle Effect" (resonance scattering of bound electrons in the magnetic field) can result in errors in the interpretation of the measured data.

c) As a consequence of the magneto-optical effects (MOE), i.e. the Faraday effect (rotation of the linear polarization within the stellar atmosphere), and the Voigt effect (phase delay within the stellar atmosphere), the damping constant Γ influences not only the tails, but generally the entire length of the Stokes parameter curves (particularly those for Q and V) within the line (see Wittman (1971)). Uncertainties and errors in Γ , which have been abundantly discussed in recent years in the literature, thus influence directly the interpretations of the measurements.

d) $|B|(\tau)$, $\gamma(\tau)$, and $\phi(\tau)$ must be assumed (and we still know very little about the solar magnetic fields as function of depth), in order to compute the form of the Stokes vector leaving the stellar atmosphere. Nevertheless, the sensitivity of the final result to the assumptions on $|B|(\tau)$, $\gamma(\tau)$, and $\phi(\tau)$ -- and thus the error limits which must be applied in comparing measured and computed Stokes parameters -- has not yet been adequately checked.

The fact that the results of the theory can only be obtained by numerical integration compels the observatories watching the solar magnetic fields to compile a kind of "Stokes-Vector Library," in which computed Stokes parameters are collected with sufficient density for all possible variations of the three field-strength components $|B|$, γ , and ϕ , and for all possible

atmosphere models and observation techniques (magnetograph splitting, etc.). It is easy to estimate that the required number of line-profile computations will be on the order of 10^4 or 10^5 . The observer then must convert his measurements (roughly 10^5 measurements can be expected for a $200'' \times 200''$ region of activity) into field components by interpolating them into this Stokes vector library.

Polarimeters

/59

As is known, devices which analyze the polarization state are called polarimeters. However, solar physicists call these devices magnetographs, although it is basically polarization which is measured, and then converted into magnetic-field components. Figs. 4 and 5 show schematically the mode of operation of a photographic and a photoelectric V-polarimeter, as well as one variant each for the operation of a photographic and a photoelectric vector polarimeter. We will not go into detailed descriptions of these self-explanatory diagrams, and refer the

reader for details to e.g. survey articles by H. von Klüber (1964) and J. W. Evans (1966). The reader is referred to the article by J. M. Beckers (1971) for further, more recent original literature. In the following, we will compare the advantages and drawbacks of the two V-polarimeters (the left half of Fig. 4 and 5):

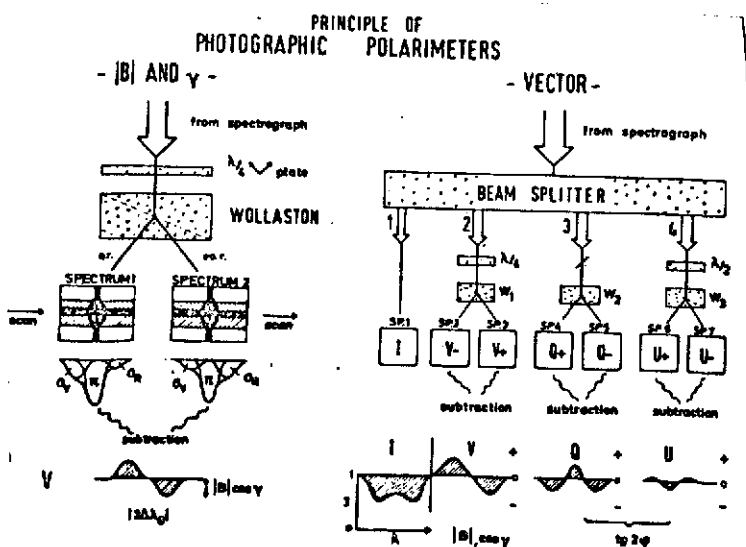


Fig. 4

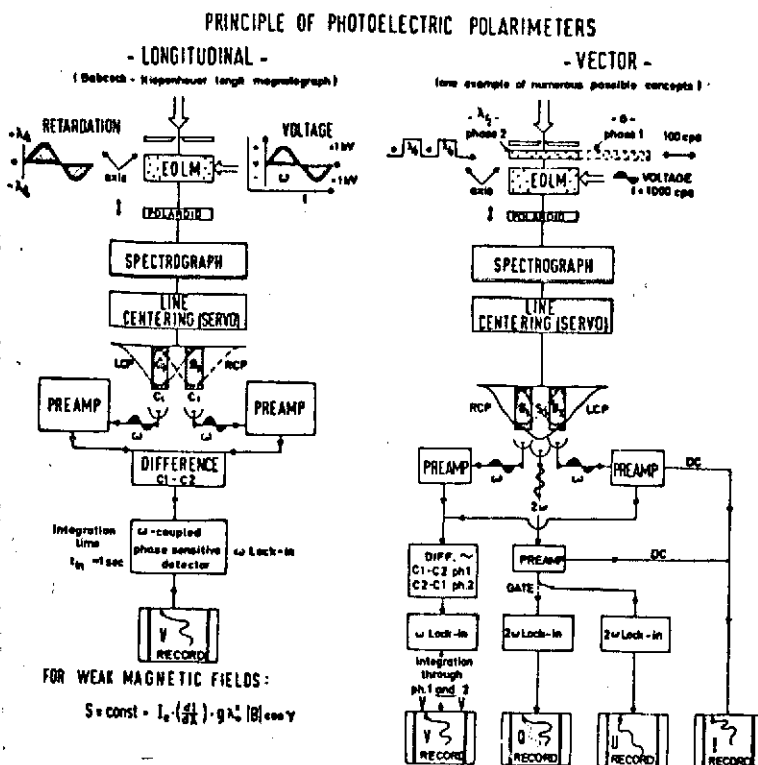


Fig. 5.

(i.e. about 33 min pure observation time) supply the information on $|B|$ and γ , and thus B_L , in a $200'' \times 200''$ region of activity. Since in this case the entire line-profile is utilized for extracting information, there are no saturation effects as in the case of the photoelectric V-polarimeters. However, much more reduction work is involved than with the Babcock-Kiepenheuer magnetograph. To study the above-mentioned $200'' \times 200''$ region with a screen unit of $(1'')^2$, a photographic precision photometry of $2 \cdot (2 \cdot 10^2)^2$ line-profiles is required. Since, on the average, there are about 10 lines per \AA in the solar spectrum, this work corresponds to a photographic photometry of an 8000\AA solar spectrum! Hence, measurements with this method on a large scale can only be made when completely automatic digital photometers are available. The work involved increases again by a factor of 3.5 if the photographic vector-polarimeter is employed for

Photographic V-Polarimeter: This device is simple, and thus inexpensive and not susceptible to break-down, and it operates quickly. With a 40-cm telescope, the exposure time on fine-grained plates is around 10 sec (Beckers and Schroter (1969) showed that, contrary to the prevalent opinion, a rms-noise of about ± 10 Gauss is obtained). With a slit height of $200''$ and a "screen line distance" of $1''$, 200 exposures

measuring the field-vector components. This is the reason why only two such devices have been utilized in solar physics (Nishi (1962), Wittman (1973)).

Photoelectric V-Polarimeter (Babcock-Kiepenheuer Magnetograph):

The large amount of electronic apparatus makes this device expensive and susceptible to malfunction; in spite of the better quantum yield of the receiver, it operates more slowly than the photographic polarimeter described above. To obtain a "rms"-noise of ± 5 Gauss, an integration time of about 1 sec per $1'' \times 1''$ measured spot is required. Thus, for a $200'' \times 200''$ region of activity, one needs $4 \cdot 10^4$ sec or about 11 hours. Information on B_L is obtained from only two subregions of the line-profile. For greater field strengths (> 1000 Gs) and reasonable widths for the collimating slits S_1 and S_2 , there are saturation effects with subsequent reduction in the signal (> 2000 Gs), so that ambiguities can arise. The problem of calibrating the magnetograph signals (converting the photoelectrically measured circular polarization into B_L) has likewise not yet been satisfactorily solved, and has resulted in the formation of an IAU Working Group in 1971. Simultaneous measurements of several observatories in this project yielded differences in B_L by up to a factor of 2. The reduction effort is certainly much smaller than in the photographic method; this essentially explains the greater "popularity" of the photoelectric polarimeter. /61

The difficulties in ascertaining the magnetic field vector (i.e. in the measurement of all Stokes parameters) is not so much in the inevitably greater complexity of the apparatus (the right halves of Figs. 4 and 5 show two possible designs for a vector polarimeter), but in the much weaker measured signal for the Stokes parameters Q and U.

Detection Limit for B_L and B_T (Evans (1966))
Longitudinal Field Component B_L ($|B| \cos \gamma$):

In the case of weak magnetic fields (only these will be considered here), the relationships (2)² show that the measured signal is the intensity change caused by the twofold splitting $2\Delta\lambda_B$. Accordingly, for the relative signal $(dI/I)_L$ (degree of circular polarization), we may write

$$(dI/I)_L = -2 v_B \cos \gamma \frac{dr}{dv} \quad v = \frac{\Delta\lambda}{\Delta\lambda_D}, \quad v_B = \frac{\Delta\lambda_B}{\Delta\lambda_D} \quad (3)$$

We assume a Gauss profile for the line-profile, and place the measuring slit at the point where (dr/dv) has its maximum, and obtain

$$(dI/I)_L = 1.7 v_B r_C \cos \gamma \quad (r_C = \text{residual penetration})$$

For the magnetically splitting line λ 5250 with $g = 3$ and $\Delta\lambda_D = 42 \text{ m \AA}$, we finally obtain:

$$(dI/I)_L = 9.4 \cdot 10^{-4} |B| \cos \gamma = 9.4 \cdot 10^{-4} \cdot B_L \quad (4)$$

Transverse Field Component B_T ($|B| \sin \gamma$)

In this case, the relationships (2) imply for the relative signal $(dI/I)_T$ (degree of linear polarization):

$$(dI/I)_T = 0.5 v_B^2 \sin^2 \gamma \frac{d^2 r}{dv^2} \quad (5)$$

With the same assumptions as above, with a measuring slit in the center of the line λ 5250, we obtain

² There is no equation (2) in original.

$$(dI/I)_T = 2.6 \cdot 10^{-7} |B|^2 \sin^2 \gamma = 2.6 \cdot 10^{-7} \cdot B_T^2 \quad (6)$$

TABLE 1

B_L, B_T	$(dI/I)_L$	$(dI/I)_T$
500 Gs	47 %	6.20 %
100 Gs	9.4 %	0.26 %
50 Gs	4.7 %	$6 \cdot 10^{-2}$ %
10 Gs	0.9 %	$2.6 \cdot 10^{-3}$ %

Since experience has shown that (even with photoelectric methods) /63 the measuring limit of 0.1% cannot be reduced, the comparison in Table 1 shows that there is little hope that the magnetic vector can be measured with greater precision than ± 100 Gauss.

Let P be the number of photons per sec incident upon 1 cm^2 of the telescope aperture, O the effective telescope aperture (= telescope surface \times quantum yield of the entire apparatus), and t the integration time.

Then the number of the light quanta composing both signals is

$$A_L = (POt) \cdot 9.4 \cdot 10^{-4} \cdot B_L \quad A_T = (POt) \cdot 2.6 \cdot 10^{-7} B_T^2 \quad (7)$$

The photon noise R is on the order of $(POt)^{1/2}$. We assume that the detection limit for both field components $B_{L, \min}$ and $B_{T, \min}$ is reached when $A/R \approx 2$. Then:

$$B_{L, \min} = 2.1 \cdot 10^3 (POt)^{-1/2} \quad B_{T, \min} = 2.8 \cdot 10^3 (POt)^{-1/4} \quad (8)$$

Now, for $\lambda \ 5250$:

$$P = 0.5 \cdot \frac{10^{17}}{4 \pi (AE)^2} \cdot F \cdot \Delta \lambda^* \cdot (I_\lambda / I_0) \quad (9)$$

Here, F is the scanning surface on the sun in cm^2 , AE = astronomical unit, $\Delta\lambda^*$ = width of the measuring slit in \AA and I_λ/I_0 = relative line intensity at the measuring point. The factor 0.5 takes into account the transparency of the earth's atmosphere.

With the empirical values: $\Delta\lambda^*_L = 0.1 \text{ \AA} \cdot (I_\lambda/I_0)_L = 0.6$ (sides of the line); $\Delta\lambda^*_T = 0.05 \text{ \AA}$, $(I_\lambda/I_0)_T = 0.4$ (center of the line), we obtain

$$B_{L, \text{Min}} = 6.4 \cdot 10^8 (Fot)^{-1/2} \quad B_{T, \text{Min}} = 1.6 \cdot 10^6 (Fot)^{-1/4} \quad (10)$$

While F (the scanning surface) and t (integration time) are free variables, O is fixed for a specific instrument.

For a 40-cm telescope (e.g. Locarno, Capri), $O = 1256 \cdot q$. The quantum yield q of a solar telescope including spectrograph and magnetograph is on the order to 10^{-3} (see also Deubner and Liedler (1969)), so that we finally obtain

$$B_{L, \text{Min}} = 6 \cdot 10^8 (Ft)^{-1/2} \quad B_{T, \text{Min}} = 1.6 \cdot 10^6 (F \cdot t)^{-1/4} \quad (11)$$

In Table 2 we have selected an integration time of 1 sec (taken from practice), and calculated the measurement limit for various measuring surfaces. By "duration" we mean the measuring time for a $200'' \times 200''$ magnetogram. In Table 3, we have required a measurement limit of $B_{T, \text{Min}} = 20$ Gauss, and calculated the required integration times and durations for various measuring surfaces.

TABLE 2

 $t = 1 \text{ sec}$

F	1" x 1"	2" x 2"	8" x 8"
$B_{L, \text{Min}}$	8 Gs	4 Gs	1 Gs
$B_{T, \text{Min}}$	185 Gs	130 Gs	64 Gs
Duration	$\sim 10^h$	2.5 h	40m

TABLE 3

 $B_{T, \text{Min}} = 20 \text{ Gs}$

F	1" x 1"	2" x 2"	8" x 8"
t	2h	30m	2m
Duration	$\sim 10^s$	$\sim 1/2^s$	20h

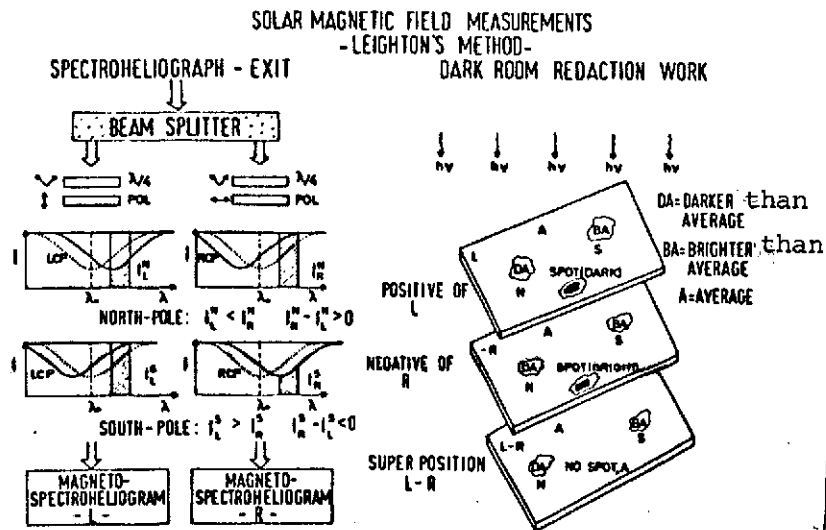


Fig. 6.

Accordingly, there is virtually no hope of measuring the field vector with a precision of ± 20 Gauss. The only parameter which we can change is θ , and this would have to be increased by 63

about 10^3 to achieve such a measurement limit with acceptable measuring times. Enlarging F beyond 8" by 8" is pointless, since then all the fine structures in the magnetic field are washed out.

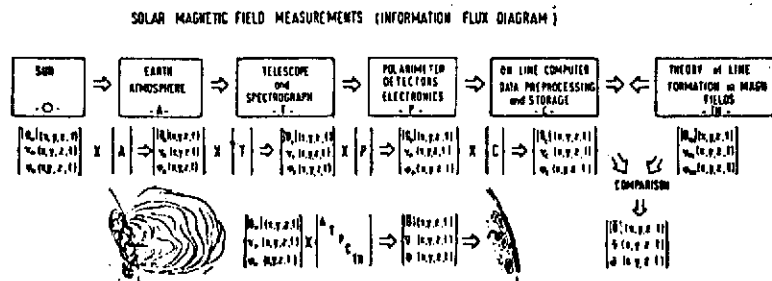


Fig. 7.

Thus, for the time being, we must content ourselves with the measurement of the longitudinal component -- outside of sunspots. Fig. 7 summarizes in the form of an information flow diagram the principal components which distort or falsify solar magnetic field measurements. The part of the diagram at the lower left is intended to draw attention to the fact that even when we have satisfactorily corrected for the influences on the matrix (A, T, P, C, TH), we have measured the magnetic field vector in a thin layer of about 100 km, compared with a characteristic magnetic-field extension of 10^3 to 10^5 km.

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